NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES

SCHOOL OF NATURAL AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BAMS	**LEVEL:** 6
COURSE CODE: LIA601S	**COURSE NAME:** LINEAR ALGEBRA
SESSION: JUNE 2023	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS: 100**

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	DR. NA CHERE
MODERATOR:	DR. DSI IIYAMBO

INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

QUESTION 1 [6]

1.1. If the nullity of the linear transformation $T: P_n \to M_{mn}$ is 3, then determine the rank of T. [3]

1.2. Prove that a square matrix A is invertible if and only if 0 is not an eigenvalue of A. [3]

QUESTION 2 [16]

Determine whether each of the following mappings is linear or not.

2.1. $T: \mathcal{F} \to \mathcal{F}$ defined by $T(f) = (f(x))^2$, where $\mathcal{F}$ is the vector space of functions on $\mathbb{R}$. [5]

2.2. $T: M_{nn} \to M_{nn}$ defined by $T(A) = AC - CA$, where C is a fixed $n \times n$ matrix. [11]

QUESTION 3 [11]

Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}2\\1\\3\end{bmatrix}$ and $T\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}1\\3\\4\end{bmatrix}$. Find $T\begin{bmatrix}a\\b\end{bmatrix}$ and use it to determine $T\begin{bmatrix}3\\2\end{bmatrix}$.

QUESTION 4 [8]

Let $\mathcal{F}$ be the vector space of functions with basis S = $\{\sin t, \ \cos t, \ e^{-2t}\}$, and let $D: \mathcal{F} \to \mathcal{F}$ be the differential operator defined by $D(f(t)) = f'(t)$. Determine the matrix $[D]_S$ representing D in the basis S.

QUESTION 5 [11]

Let $L: \mathbb{R}^4 \to \mathbb{R}^3$ be the linear mapping defined by $L\left(\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}\right) = \begin{bmatrix}x_1 - x_2 + x_3 + x_4\\2x_1 - 2x_2 + 3x_3 + 4x_4\\3x_1 - 3x_2 + 4x_3 + 5x_4\end{bmatrix}$.

Find the basis and the dimension of the image of L.

QUESTION 6 [11]

Consider the bases $B = \{1 + x + x^2, x + x^2, x^2\}$ and $C = \{1, x, x^2\}$ of P_2.

6.1. Find the change of basis matrix $P_{B \leftarrow C}$ from C to B. [8]

6.2. Use the result in part (6.1) to compute $[p(x)]_B$ where $p(x) = 2 + x - 3x^2$. [3]

QUESTION 7 [26]

Consider $A = \begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix}$.

7.1. Write down the characteristic polynomial $P(\lambda)$ of A and use this to find the eigenvalues of A. [6]

7.2. Find the eigenspaces corresponding to the eigenvalues of A. [17]

7.3. Is A diagonalizable ? If so, find an invertible matrix P that diagonalizes A. [3]

QUESTION 8 [11]

Find an orthogonal change of variables that eliminates the cross-product term in the quadratic form $q(x_1, x_2, x_3) = 3x_1^2 + 2x_3^2 + 4x_1x_2$ and express q in terms of the new variables.

END OF FIRST OPPORTUNITY EXAMINATION QUESTION PAPER